

A self-consistency obstruction to internally antisymmetric nonunitary triplet pairing

Henry Sheehy^{1,2}

¹*School of Physical Sciences, University of Kent, Canterbury CT2 7NH, United Kingdom*

²*QuantaLumin*

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Internally antisymmetric nonunitary triplet (INT) pairing has been proposed as a way to reconcile time-reversal-symmetry breaking, multigap thermodynamics, and apparently nodeless superconductivity in LaNiX₂-inspired multiorbital models. The proposal is attractive because an equal-spin triplet can have even spatial parity when the pair wave function is antisymmetric in an orbital index. Here we identify a self-consistency obstruction in this route. Starting from a two-orbital Hubbard-Kanamori interaction, we derive the local INT channel, impose the corresponding nonunitary Bogoliubov-de Gennes gap, and then relax competing anomalous fields, with compact normal-field feedback tests, in toy Hamiltonians and survey material-derived inputs. The imposed state has the expected split $\Delta\Delta^\dagger$, spin-resolved density-of-states asymmetry, and condensate spin polarization. The self-consistency tests are more restrictive. A scalar INT attraction can sustain the local triplet amplitude but does not by itself select the nonunitary imbalance between equal-spin components; a robust low-energy spectrum also requires spin-orbital texture that keeps INT-paired partner states inside the Fermi subspace. The central result is therefore not a material exclusion theorem but a mechanism statement: scalar pairing attraction and nonunitary pair-spin selection are distinct mean-field directions. Current spin-orbit-coupled LaNiC₂ and LaNiGa₂ Wannier survey Hamiltonians provide a consistency check and also do not select a robust nonunitary INT branch, but those inputs are retained only at survey level. A production material-realistic LaNiX₂ theory is a separate follow-up program rather than a prerequisite for the obstruction demonstrated here.

I. INTRODUCTION

Muon spin relaxation experiments report time-reversal-symmetry breaking (TRSB) in both noncentrosymmetric LaNiC₂ and centrosymmetric LaNiGa₂ [1, 2]. At the same time, thermodynamic and penetration-depth measurements point to multigap and largely nodeless superconductivity [3, 4]. This combination is difficult to accommodate in a conventional single-band triplet picture, where nonunitary triplet states normally carry odd spatial parity and tend to have nodal structures constrained by crystal symmetry [5]. Both compounds are orthorhombic rather than tetragonal, so the material-derived calculations below do not assume four-fold crystal-rotation symmetry.

The internally antisymmetric nonunitary triplet proposal resolves the exchange-symmetry problem by using an additional orbital label [2, 3, 6]. The Cooper pair is spin triplet and equal-spin, but its orbital part is antisymmetric, allowing an even-parity local gap. If the two equal-spin amplitudes are unequal, the condensate is nonunitary and can carry a spin-polarized TRSB signature. The scenario is therefore a natural bridge between the experimental facts: two gaps, weak spontaneous fields, and no obligatory line nodes.

The novelty of this work is not the symmetry classification of nonunitary superconductivity, which is well established and has recently been generalized to multiorbital complex materials [5, 7]. Nor is it the observation that Hund exchange can favor interorbital triplet channels; that mechanism and its competition with spin-fluctuation pairing have been analyzed in broader multiorbital settings [8, 9]. Related work also shows that

competing electronic orders can promote nonunitary multiorbital superconductivity in other microscopic models [10]. Our contribution is instead a self-consistency audit of the specific LaNiX₂-motivated INT route. We ask which parts of the phenomenology survive when the gap and normal feedback are allowed to relax, and whether a scalar local INT attraction is enough to choose the nonunitary branch.

The question addressed here is narrower than the phenomenology. We ask whether the INT state follows as a self-consistent solution of a local two-orbital interaction, using material-derived LaNiX₂ normal-state Hamiltonians only as a survey-level extension of the same diagnostic. This is a stricter test than imposing the gap. An imposed nonunitary order parameter may reproduce the desired signatures, but a microscopic derivation must also show that the freely relaxed anomalous and normal fields select the nonunitary branch over singlet, unitary triplet, and mixed competitors. The material-derived calculations are intentionally demoted relative to this mechanism question: the reduced-basis scans include compact normal feedback, while the full-basis Wannier scans are deliberately quick anomalous-first tests rather than production unrestricted full-Wannier Hartree-Fock-Gor'kov minimizations.

Our answer is cautious and negative at the level tested here. The local INT channel is well defined, and the imposed INT state gives the expected nonunitary diagnostics. However, the present self-consistent toy calculations relax toward unitary or non-INT branches unless an explicit nonunitary bias or spin-polarization feedback is supplied. The current material-derived LaNiC₂ and LaNiGa₂ spin-orbit-coupled Wannier survey Hamil-

tonians likewise do not stabilize a robust nonunitary INT branch. Because the LaNiGa₂ SOC input has only convergence-limited segmented direct-band validation and a large Wannier spread, this should not be read as a final exclusion theorem. The useful conclusion is the obstruction itself: under the tested local Hubbard-Kanamori-like assumptions, INT is a coherent constrained mechanism, but scalar attraction selects amplitude rather than spontaneous nonunitarity. This obstruction is also constructive. It says that a microscopic TRSB theory cannot be specified by naming only an attractive superconducting channel; it must separately explain how amplitude survives and how the time-reversal-odd coordinate is selected without an external source. That separation motivates subsequent loop-supercurrent constructions, where amplitude support and phase/current selection can be engineered and tested as distinct self-consistency ingredients. We therefore treat the material-realistic LaNiX₂ program as future work and keep the present claim bounded to the mechanism audit, the imposed-state diagnostics, and the negative survey tests.

II. MODEL AND INT CHANNEL

We use a two-orbital spinor

$$\hat{\Psi}_{\mathbf{k}} = (\hat{c}_{\mathbf{k}a\uparrow} \ \hat{c}_{\mathbf{k}a\downarrow} \ \hat{c}_{\mathbf{k}b\uparrow} \ \hat{c}_{\mathbf{k}b\downarrow})^T. \quad (1)$$

The minimal normal-state toy Hamiltonian is

$$h_0(\mathbf{k}) = \xi_+(\mathbf{k})\tau_0\sigma_0 + \xi_-(\mathbf{k})\tau_z\sigma_0 + v(\mathbf{k})\tau_x\sigma_0 + \lambda\tau_y\sigma_z, \quad (2)$$

where τ_i and σ_i act in orbital and spin space. The two-orbital toy figures use the two-dimensional dispersion

$$\begin{aligned} \xi_+(\mathbf{k}) &= -2t(\cos k_x + \cos k_y) - \mu, \\ \xi_-(\mathbf{k}) &= 2ts, \\ v(\mathbf{k}) &= v_0 + 2v_1(\cos k_x + \cos k_y), \\ \xi_{a,b}(\mathbf{k}) &= \xi_+(\mathbf{k}) \pm \xi_-(\mathbf{k}). \end{aligned} \quad (3)$$

Thus t fixes the nearest-neighbor hopping scale, μ is the chemical potential measured in that scale, s is the orbital splitting in units of t , and v_0, v_1 are local and extended interorbital hybridization amplitudes. The baseline longitudinal spin-orbit term in Eq. (2) is $\lambda \equiv \lambda_z$. Projection-repair scans additionally allow transverse texture terms such as $\lambda_x\tau_y\sigma_x$; these are switched on only for the diagnostic repair tests. The local interaction is the two-orbital Hubbard-Kanamori form [8, 11],

$$\hat{H}_{\text{int}} = \hat{H}_U + \hat{H}_{U'} + \hat{H}_J + \hat{H}_P, \quad (4)$$

with

$$\hat{H}_U = U \sum_i \sum_{m=a,b} \hat{n}_{im\uparrow} \hat{n}_{im\downarrow}, \quad (5)$$

$$\hat{H}_{U'} = U' \sum_i \sum_{\sigma,\sigma'} \hat{n}_{ia\sigma} \hat{n}_{ib\sigma'}, \quad (6)$$

$$\hat{H}_J = -J_H \sum_i \sum_{\sigma,\sigma'} \hat{c}_{ia\sigma}^\dagger \hat{c}_{ib\sigma'}^\dagger \hat{c}_{ib\sigma} \hat{c}_{ia\sigma'}, \quad (7)$$

$$\begin{aligned} \hat{H}_P = J_P \sum_i & \left(\hat{c}_{ia\uparrow}^\dagger \hat{c}_{ia\downarrow}^\dagger \hat{c}_{ib\downarrow} \hat{c}_{ib\uparrow} \right. \\ & \left. + \hat{c}_{ib\uparrow}^\dagger \hat{c}_{ib\downarrow}^\dagger \hat{c}_{ia\downarrow} \hat{c}_{ia\uparrow} \right), \end{aligned} \quad (8)$$

where $\hat{n}_{im\sigma} = \hat{c}_{im\sigma}^\dagger \hat{c}_{im\sigma}$. The sign convention is that positive U, U', J_H , and J_P denote repulsive local interaction parameters in the microscopic Hamiltonian, while a negative pairing vertex is attractive in the linearized gap equation. For two electrons in different orbitals, the density term contributes U' . Hund exchange lowers the spin-triplet multiplet relative to the interorbital singlet; in the equal-spin triplet sector the local scale is therefore

$$V_{\text{INT}} = U' - J_H. \quad (9)$$

Equivalently, with the ordering above,

$$\langle a\sigma, b\sigma | \hat{H}_{U'} + \hat{H}_J | a\sigma, b\sigma \rangle = U' - J_H, \quad (10)$$

which defines the channel scale used in Eq. (9). Hund exchange lowers the equal-spin interorbital triplet channel relative to competing interorbital channels, but this does not by itself imply an attractive superconducting instability. For realistic repulsive parameters the INT sector can be the least repulsive triplet descendant without being an instability unless the effective vertex is attractive or renormalized.

The inverse scalar pairing susceptibility is defined by the channel kernel

$$\mathcal{K}_\alpha(T, J_H) = \chi_\alpha^{-1}(T) + \mathbf{U}_\alpha(J_H), \quad (11)$$

where χ_α is the normal-state pair susceptibility projected onto channel α . Equivalently, Fig. 1 plots the dimensionless eigenvalues $\lambda_\alpha(T, J_H) = -\mathbf{U}_\alpha(J_H)\chi_\alpha(T)$, for which $\lambda_\alpha = 1$ marks the scalar linearized instability condition. For that plot we hold the toy-model susceptibilities fixed at $T = 0.05$ and sweep the local Kanamori interaction vector using

$$\begin{aligned} \mathbf{U}_{aa} &= \mathbf{U}_{bb} = U, \\ \mathbf{U}_{ab,S=0} &= U' + J_H, \\ \mathbf{U}_{\text{INT}} &= U' - J_H, \end{aligned} \quad (12)$$

with $U = 2$ and $U' = 1$ in the plotted units. The plotted eigenvalues show the same sign convention from the instability side: Hund exchange lowers the equal-spin triplet interaction scale, which raises its plotted pairing eigenvalue relative to interorbital singlet competitors. The

conservative bare hierarchy nevertheless remains repulsive until $J_H > U'$ or an effective attractive INT vertex is supplied.

The imposed local INT gap is

$$\Delta_{\text{INT}} = \begin{pmatrix} 0 & 0 & \Delta_{\uparrow\uparrow} & 0 \\ 0 & 0 & 0 & \Delta_{\downarrow\downarrow} \\ -\Delta_{\uparrow\uparrow} & 0 & 0 & 0 \\ 0 & -\Delta_{\downarrow\downarrow} & 0 & 0 \end{pmatrix}. \quad (13)$$

Equivalently,

$$\Delta_{\text{INT}} = \underbrace{i\tau_y}_{\text{orbital antisymmetric}} \otimes \underbrace{(\Delta_{\uparrow\uparrow}P_{\uparrow} + \Delta_{\downarrow\downarrow}P_{\downarrow})}_{\text{equal-spin sectors}}, \quad (14)$$

where $P_{\uparrow} = (\sigma_0 + \sigma_z)/2$ and $P_{\downarrow} = (\sigma_0 - \sigma_z)/2$. The minus signs in Eq. (13) are the entries of the antisymmetric orbital tensor $i\tau_y$. The state is nonunitary when

$$id \times d^* \neq 0, \quad (15)$$

or equivalently when $\Delta_{\text{INT}}\Delta_{\text{INT}}^{\dagger} \not\propto I$ in the paired subspace [5, 12]. Here the paired subspace means the image of the local INT matrix on the equal-spin interorbital sector; in practice we compare the nonzero eigenvalues of $\Delta_{\text{INT}}\Delta_{\text{INT}}^{\dagger}$ on that support. In the Bogoliubov-de Gennes (BdG) problem,

$$\mathcal{H}_{\text{BdG}}(\mathbf{k}) = \begin{pmatrix} h_0(\mathbf{k}) + \phi(\mathbf{k}) & \Delta(\mathbf{k}) \\ \Delta^{\dagger}(\mathbf{k}) & -h_0^T(-\mathbf{k}) - \phi^T(-\mathbf{k}) \end{pmatrix}, \quad (16)$$

both the normal field ϕ and anomalous field Δ must be updated from the density matrix and Gor'kov contraction in a self-consistent calculation. For a local interaction written with an antisymmetrized vertex

$$\hat{H}_{\text{int}} = \frac{1}{2} \sum_{\ell_1 \ell_2 \ell_3 \ell_4} \mathbf{U}_{\ell_1 \ell_2; \ell_3 \ell_4} \hat{c}_{\ell_1}^{\dagger} \hat{c}_{\ell_2}^{\dagger} \hat{c}_{\ell_4} \hat{c}_{\ell_3}, \quad (17)$$

where ℓ combines orbital and spin, we use $\rho_{\ell\ell'} = \langle \hat{c}_{\ell'}^{\dagger} \hat{c}_{\ell} \rangle$ and $\chi_{\ell\ell'} = \langle \hat{c}_{\ell'} \hat{c}_{\ell} \rangle$. The mean-field decoupling has the compact form

$$\hat{H}_{\text{MF}} = \sum_{\ell\ell'} \phi_{\ell\ell'} \hat{c}_{\ell}^{\dagger} \hat{c}_{\ell'} + \frac{1}{2} \sum_{\ell\ell'} \left(\Delta_{\ell\ell'} \hat{c}_{\ell}^{\dagger} \hat{c}_{\ell'}^{\dagger} + \text{h.c.} \right) - C_{\text{MF}}, \quad (18)$$

with $\phi[\rho]$ generated by the particle-hole Hartree/Fock contraction of $\mathbf{U}$ and $\Delta[\chi]$ generated by the particle-particle contraction. With the antisymmetry and sign convention absorbed into $\mathbf{U}$, this is schematically

$$\phi_{\ell_1 \ell_3} = \sum_{\ell_2 \ell_4} \mathbf{U}_{\ell_1 \ell_2; \ell_3 \ell_4} \rho_{\ell_4 \ell_2}, \quad (19)$$

$$\Delta_{\ell_1 \ell_2} = \sum_{\ell_3 \ell_4} \mathbf{U}_{\ell_1 \ell_2; \ell_3 \ell_4} \chi_{\ell_3 \ell_4}. \quad (20)$$

Equations (20) give the mean-field self-consistency fields used below: anomalous-only updates keep the $\Delta[\chi]$ part, density-density Hartree feedback keeps only diagonal direct terms in $\phi[\rho]$, and compact unrestricted Kanamori feedback keeps local Hartree/Fock normal contractions together with the anomalous update.

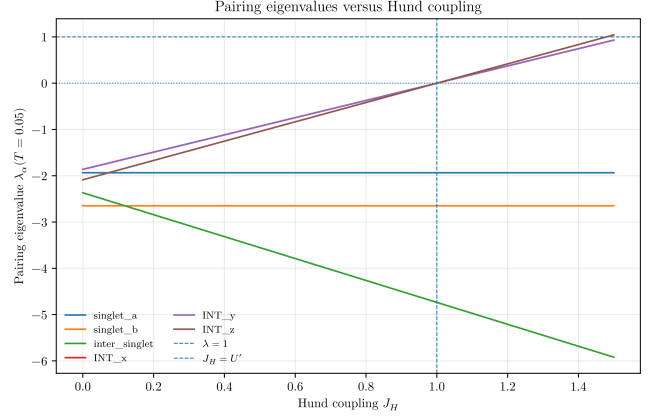


FIG. 1. Linearized local-channel diagnostic. The plotted eigenvalues are $\lambda_{\alpha} = -\mathbf{U}_{\alpha}\chi_{\alpha}$, equivalently the scalar inverse-susceptibility kernel $\mathcal{K}_{\alpha} = \chi_{\alpha}^{-1} + \mathbf{U}_{\alpha}$ crossing zero at $\lambda_{\alpha} = 1$. Hund exchange lowers the equal-spin triplet interaction scale $U' - J_H$, which raises the corresponding plotted instability eigenvalue relative to interorbital singlet competitors. The conservative bare hierarchy remains repulsive unless $J_H > U'$ or an effective attractive INT vertex is supplied.

III. FERMI-SURFACE PROJECTION

Channel selection is not enough. The gap that matters for weak pairing is the projection of the local matrix into the Fermi subspace,

$$\Delta_F(\mathbf{k}) = P_F(\mathbf{k})\Delta_{\text{INT}}P_F^T(-\mathbf{k}). \quad (21)$$

Here $P_F(\mathbf{k})$ is the orthogonal projector onto the retained low-energy normal-state eigenvectors, either the selected weak-pairing Fermi-window states in the toy scan or the four near-Fermi states in the reduced material benchmark. Equation (21) is the central weak-pairing test: a large local gap matrix is ineffective if it mostly maps a Fermi-level state to a partner outside the low-energy subspace. In the minimal Hamiltonian of Eq. (2), the local INT matrix can be large while its same-Fermi-subspace projection is small. The INT operator then connects a low-energy state mostly to a partner outside the Fermi window, producing a near-nodal spectrum even when the linearized INT channel is favorable.

This obstruction can be exposed analytically by keeping only local spin-orbit terms

$$h_{\text{rep}}(\mathbf{k}) = \xi(\mathbf{k})\tau_0\sigma_0 + \tau_y(\lambda_x\sigma_x + \lambda_z\sigma_z). \quad (22)$$

Here $\xi(\mathbf{k})$ is a scalar dispersion common to the four internal states. The matrix τ_y is the imaginary orbital-antisymmetric hybridization structure, and $\lambda_x\sigma_x + \lambda_z\sigma_z$ gives the spin-orbital texture carried by that hybridization. In the convention of Eq. (13), $\lambda_z\sigma_z$ is longitudinal with respect to the chosen $\uparrow/\downarrow$ spin quantization axis, while $\lambda_x\sigma_x$ is transverse. This terminology is basis and gap-convention dependent, but it is useful for the projec-

tion argument below. The helicity projectors are

$$P_\nu = \frac{1}{2} \left[1 + \nu \tau_y \frac{\lambda_x \sigma_x + \lambda_z \sigma_z}{\sqrt{\lambda_x^2 + \lambda_z^2}} \right], \quad \nu = \pm. \quad (23)$$

We use the equal-spin triplet convention

$$\Delta_{\uparrow\uparrow} = -d_x + id_y, \quad \Delta_{\downarrow\downarrow} = d_x + id_y. \quad (24)$$

Thus the Δ_x entering the projection formula denotes the d_x component,

$$\Delta_x \equiv d_x = \frac{\Delta_{\downarrow\downarrow} - \Delta_{\uparrow\uparrow}}{2}. \quad (25)$$

For the simplified Hamiltonian in Eq. (22), the fixed INT convention above, and the d_x component just defined, the nonzero singular values of $P_\nu \Delta_{\text{INT}} P_\nu^T$ scale as

$$s_\nu = \left| \Delta_x \frac{\lambda_x}{\sqrt{\lambda_x^2 + \lambda_z^2}} \right|. \quad (26)$$

A purely longitudinal spin-orbit texture, $\lambda_x = 0$, therefore leaves the same-helicity INT projection zero in this limit. A transverse spin-orbital component immediately repairs the weak-pairing gap because the normal-state eigenvectors then contain the spin-orbital rotation needed for Δ_{INT} to act within one helicity Fermi sheet. Thus the projection rule is not a statement that larger scalar attraction fixes the spectrum; it states that a purely longitudinal texture fails to repair the weak-pairing projection, while a transverse component can keep the INT-paired partner in the low-energy subspace. In the retained repair scan, the displayed transverse case with $\lambda_x = 0.6$ gives a finite Fermi-surface minimum gap, while longitudinal tuning alone leaves the projected INT weight near zero. The scalar diagnostics plotted in Fig. 2 are evaluated on normal-state Fermi-window momenta,

$$\mathcal{F}_{\delta_F} = \{\mathbf{k} : \min_n |\xi_n(\mathbf{k})| < \delta_F\}, \quad (27)$$

$$E_{\min}^{\text{FS}} = \min_{\mathbf{k} \in \mathcal{F}_{\delta_F}} \min_{E_m(\mathbf{k}) > 0} E_m(\mathbf{k}), \quad (28)$$

$$s_{\max}(\mathbf{k}) = \sigma_{\max} [P_F(\mathbf{k}) \Delta_{\text{INT}} P_F^T(-\mathbf{k})], \quad (29)$$

$$w_{\text{low}}(\mathbf{k}) = \frac{\|P_F(\mathbf{k}) \Delta_{\text{INT}} P_F^T(-\mathbf{k})\|_F}{\|P_F(\mathbf{k}) \Delta_{\text{INT}} U^*(-\mathbf{k})\|_F}. \quad (30)$$

Here ξ_n are normal-state band energies, E_m are positive BdG quasiparticle energies, $\sigma_{\max}$ is the largest singular value, $U(-\mathbf{k})$ is the full normal-state eigenvector matrix, and δ_F is the numerical Fermi-window cutoff. The ratio w_{low} asks what fraction of the pairing amplitude from retained Fermi states lands back in retained Fermi states rather than leaking to high-energy partners.

IV. IMPOSED NONUNITARY DIAGNOSTICS

The imposed INT state has the expected algebraic and spectral signatures. The nonunitary condition is

not $\Delta\Delta^\dagger = 1$ or $\Delta\Delta^\dagger \neq 1$; it is that $\Delta\Delta^\dagger$ is not proportional to the identity in the paired subspace. The eigenvalues of $\Delta_{\text{INT}}\Delta_{\text{INT}}^\dagger$ split when $|\Delta_{\uparrow\uparrow}| \neq |\Delta_{\downarrow\downarrow}|$, and the BdG density of states becomes spin resolved. These are necessary checks: they show that the implemented gap is genuinely nonunitary and that the code path detects the intended spin-polarized TRSB-like observables. They are not, by themselves, a calibrated spontaneous-field, orbital-current, or μSR -linewidth calculation. The spin-resolved density of states in Fig. 4 is computed from the electron block of the retarded BdG Green function after the same local impurity/Dyson construction used in the code,

$$N_s(\omega; \mathbf{r}) = -\frac{1}{\pi} \Im \text{Tr} [\Pi_s G_{ee}^R(\mathbf{r}, \mathbf{r}; \omega + i\eta)], \quad (31)$$

where Π_s projects onto spin $s = \uparrow, \downarrow$ in the local electron basis and η is the Lorentzian broadening. Equivalently, before the impurity Dyson step this is the spectral sum $N_s(\omega) = N_k^{-1} \sum_{\mathbf{k}n} \langle n\mathbf{k} | \mathcal{P}_{e,s} | n\mathbf{k} \rangle \delta_\eta(\omega - E_{n\mathbf{k}})$ with $\delta_\eta(x) = \eta/[\pi(x^2 + \eta^2)]$.

The distinction between imposed diagnostics and phase selection is central. In a self-consistent calculation the fixed point

$$\mathcal{H}_{\text{BdG}}[\rho^{(n)}, \chi^{(n)}] \rightarrow \{\rho^{(n+1)}, \chi^{(n+1)}\} \rightarrow \{\phi^{(n+1)}, \Delta^{(n+1)}\} \quad (32)$$

must be allowed to choose among competing anomalous structures. The imposed nonunitary branch is a useful probe, but the relevant ground-state test is the free-energy ordering of relaxed singlet, unitary INT, nonunitary INT, mixed, and random seeds. The quantity ranked in the code is

$$\Omega_{\text{MF}} = \Omega_{\text{spec}}[\mathcal{H}_{\text{BdG}}] + C_{\text{pp}}[\Delta, \chi] + C_{\text{ph}}[\phi, \rho], \quad (33)$$

where Ω_{spec} is the quasiparticle grand potential and the constants remove the particle-particle and particle-hole double counting introduced by Eq. (18). For finite temperature the spectral part is evaluated from the positive BdG eigenvalues as

$$\Omega_{\text{spec}} = -T \left\langle \sum_{E_{n\mathbf{k}} > 0} \log \left[2 \cosh \left(\frac{E_{n\mathbf{k}}}{2T} \right) \right] \right\rangle_{\mathbf{k}}, \quad (34)$$

up to the common normal-state reference used in branch comparisons. Tables below report $\Omega_{\text{MF}} - \Omega_N$; a positive value means that the relaxed superconducting seed remains above the normal reference in the quick scan.

V. TOY SELF-CONSISTENCY

In the toy fixed-point tests, scalar INT attraction can sustain local interorbital triplet pairing. However, when the two equal-spin components are allowed to relax without an explicit nonunitary bias, the solution tends toward a unitary INT branch rather than a robust imbalance. Adding minimal Kanamori normal feedback does

Best projection repair: lam_x, strength=-0.6000

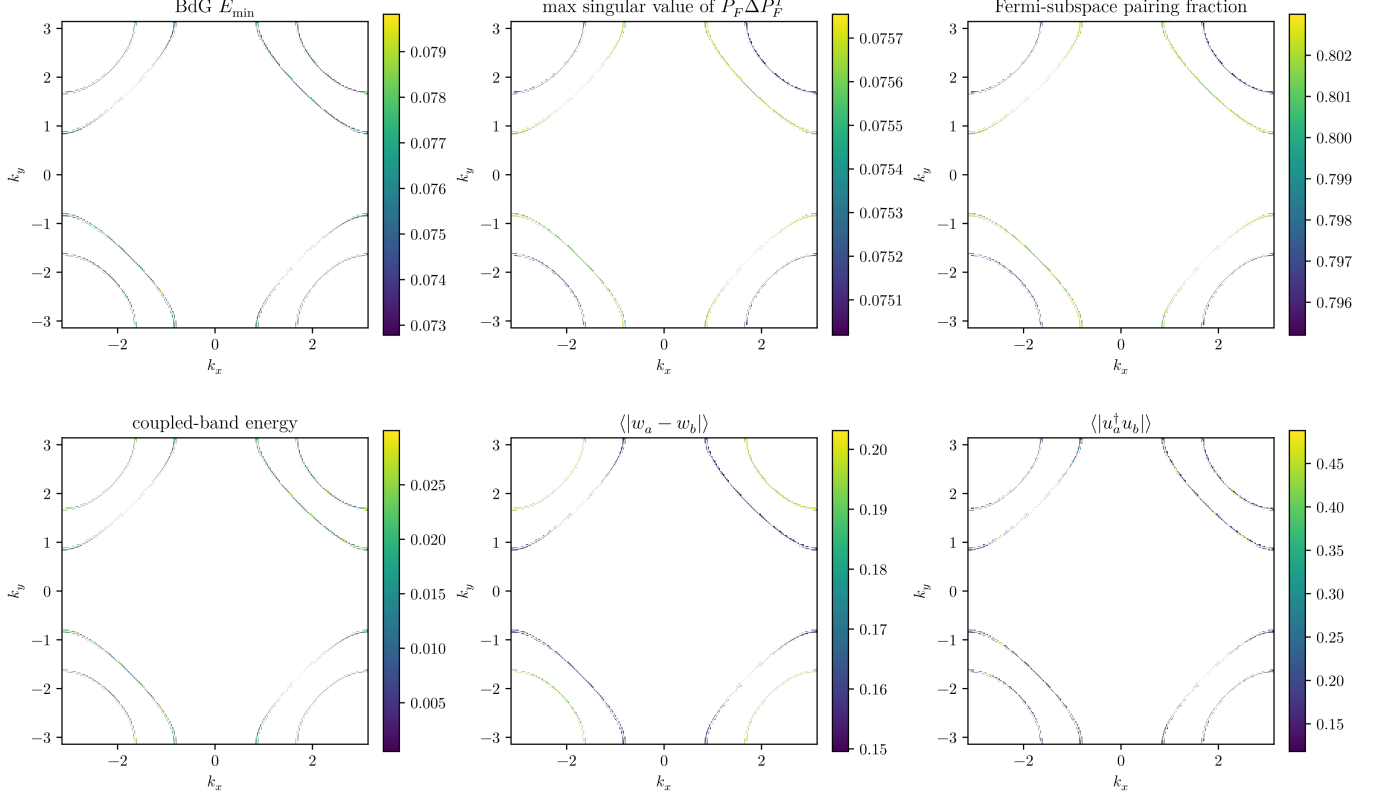


FIG. 2. Why a favorable local INT vertex is not enough. The weak-pairing gap is controlled by the projected matrix $\Delta_F(\mathbf{k}) = P_F(\mathbf{k})\Delta_{\text{INT}}P_F^T(-\mathbf{k})$, not by the local INT matrix alone. The panels report the Fermi-window diagnostics in Eqs. (21) and (30): the minimum BdG gap, the largest projected singular value, the retained-subspace weight, and associated normal-state texture measures. In the minimal toy model the INT operator mostly pairs a Fermi-level state with a partner outside the retained low-energy subspace, leaving a weak projected gap and small retained weight. A transverse spin-orbital texture repairs this by keeping the INT-paired partners in the same Fermi subspace, whereas increasing the scalar attraction alone does not.

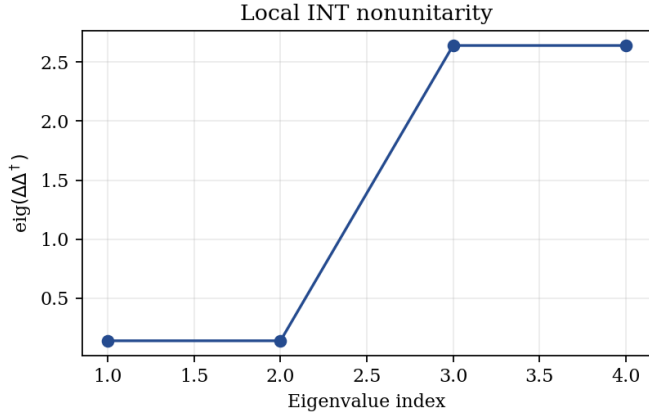


FIG. 3. Nonunitarity of the imposed INT gap. Unequal equal-spin components split the eigenvalues of $\Delta\Delta^\dagger$, the direct algebraic signature of a nonunitary triplet state.

not change this qualitative outcome. The result is not a convergence failure: the iterations converge, but the

nonunitary component is not freely selected.

The distinction can be stated without reference to any numerical detail. For the two equal-spin INT amplitudes define

$$A = |\Delta_{\uparrow\uparrow}|^2 + |\Delta_{\downarrow\downarrow}|^2, \quad Q = |\Delta_{\uparrow\uparrow}|^2 - |\Delta_{\downarrow\downarrow}|^2. \quad (35)$$

Here A measures the scalar INT amplitude, while Q measures the pair-spin or nonunitary imbalance. A schematic local free energy near the INT sector has the form

$$F_{\text{INT}} = rA + uA^2 + \kappa Q^2 + \dots, \quad (36)$$

where a scalar attractive INT vertex primarily drives r negative. It does not by itself require $\kappa < 0$. Thus a self-consistent solution may have $A > 0$ but $Q = 0$: a unitary INT state rather than a spontaneous nonunitary one. The obstruction tested below is precisely whether the microscopic feedback generates the missing negative stiffness, or equivalent nonlinear selector, in the Q direction.

The natural positive-control toy model is therefore a repaired INT normal state, Eq. (22) with finite transverse

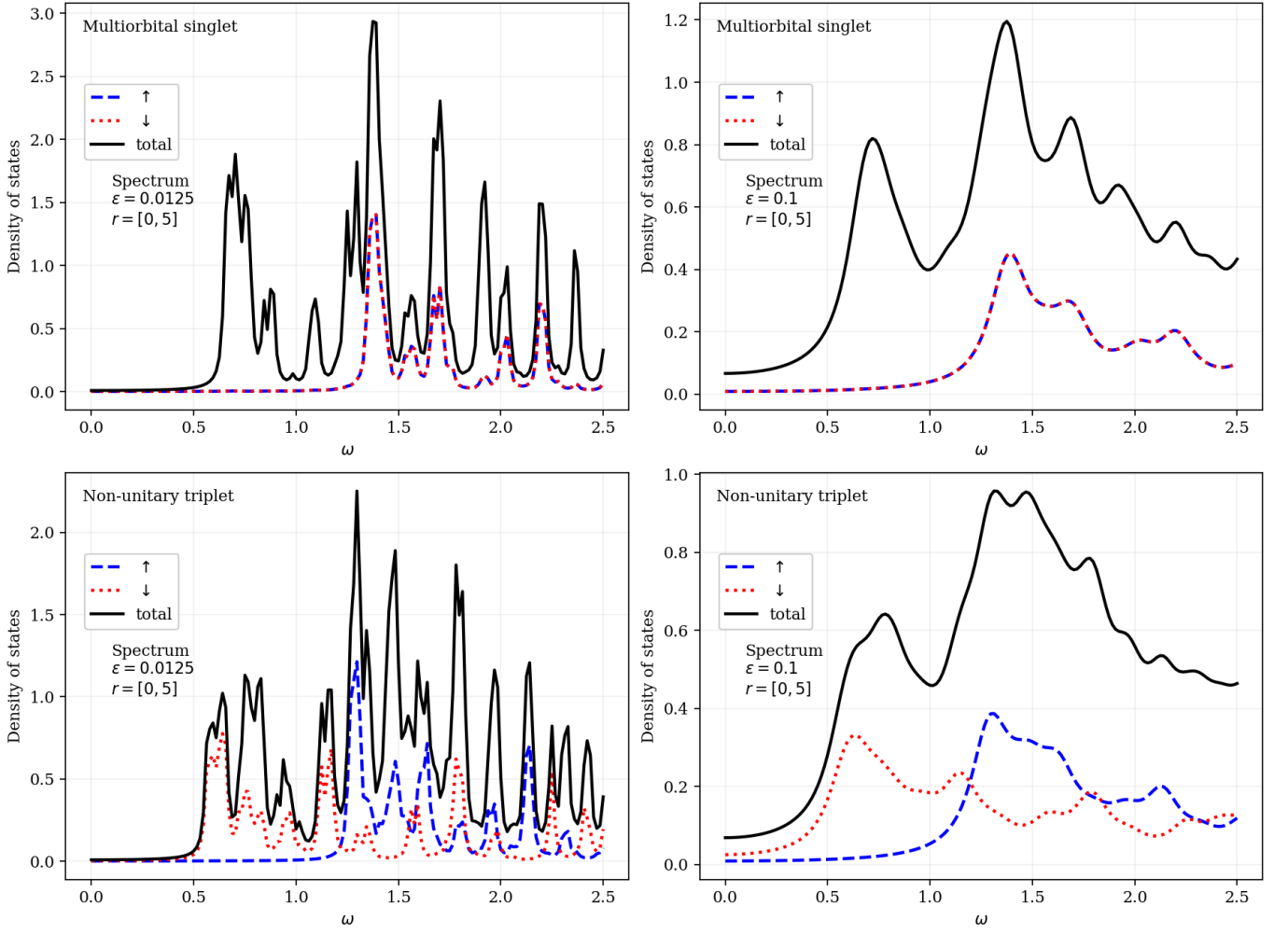


FIG. 4. Spin-resolved density of states for the imposed INT ansatz. The plotted curves use the local spectral function defined in Eq. (31) at the coordinate and broadenings shown in each panel. The nonunitary gap produces a spin-resolved spectral asymmetry, but this imposed diagnostic does not by itself prove self-consistent phase selection.

λ_x , supplemented by an explicit source-free pair-spin selector. At the level of Eq. (36) this means driving $r < 0$ while also making the effective stiffness of Q negative, for example through a term $-KQ^2$ with $K > \kappa$. Such a control is useful only if the two signs of Q remain degenerate: it should select the two time-reversed $d_x \pm id_y$ domains without applying a Zeeman field or fixed chiral source. The QuLab positive-control kernel implements exactly this solver check by leaving the normal seed at zero and projecting finite INT seeds onto the circular $d_x \pm id_y$ sector. It is a sign and HFG-plumbing benchmark, not a microscopic material mechanism.

Table I summarizes the mechanism-level message. The important separation is not between “toy” and “material”, but between amplitude support and a genuine selector for Q .

Equations (35) and (36) summarize the interpretation of Fig. 5. A microscopic stabilization of nonunitarity requires more than a scalar attractive INT vertex. A minimal phenomenological route would be a

TABLE I. Compact mechanism audit for the INT selection problem. The positive-control row shows that the solver can carry nonunitary domains when a selector is supplied; it is not microscopic evidence for a material mechanism.

Mechanism	Outcome
Scalar INT attraction	finite A , $Q \rightarrow 0$ obstruction
Projection repair	restores INT weight; required texture
Kanamori feedback	unitary/mixed; no selector
Stoner-like feedback	benchmark decays in this test
Designed $-KQ^2$ control	degenerate $d_x \pm id_y$ positive control
Material response proxies	mixed/unitary/normal survey negative

normal spin-polarization feedback term, for example a Stoner-like energy lowering for condensate spin polarization $M \propto id \times d^*$. This mechanism is distinct from simply increasing the scalar INT attraction: the attraction controls the total pairing amplitude, while a separate exchange or spin-polarization feedback can control the

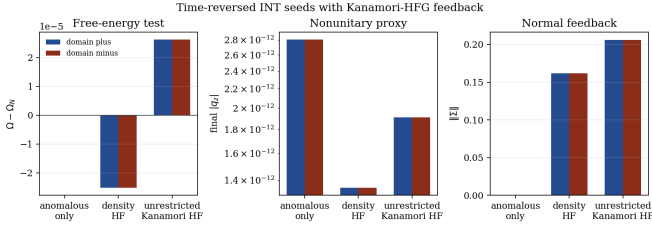


FIG. 8. Time-reversed INT seeds with compact Kanamori-HFG feedback. The anomalous-only, density-density Hartree/Fock, and unrestricted Kanamori Hartree/Fock updates are run from two nonunitary seeds related by time reversal. The density and unrestricted feedback modes generate finite normal fields, but the final $|q_z| = |(id \times d^*)_z|$ remains at numerical floor for the tested tiny parameters. This benchmark rules out the simplest local Kanamori-HFG feedback implementation as the missing mechanism in the toy model, without excluding broader material-scale exchange effects.

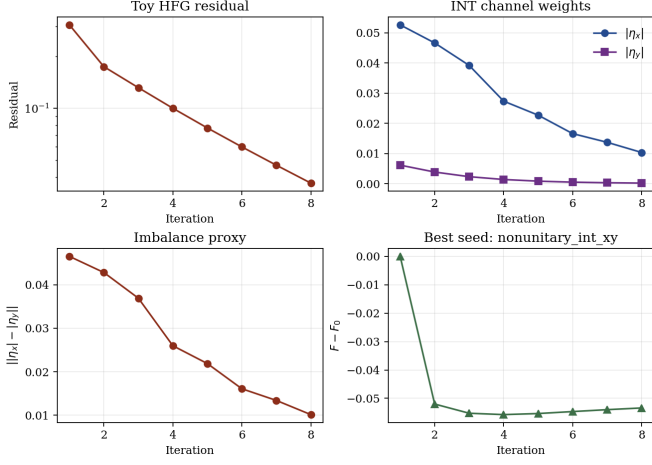


FIG. 9. Representative toy HFG convergence trace. The residual, INT channel weights, imbalance proxy, and free-energy change converge in the same calculation used for the phase-selection benchmark. Convergence alone does not imply nonunitary phase selection; in this representative run the imbalance proxy remains small compared with the relaxed pairing scale.

toy calculations is immediately removed by the available LaNiX_2 spin-orbit texture and Wannier basis. The survey inputs used here are spin-orbit-coupled LaNiX_2 Wannier Hamiltonians generated in the Kent/Icarus workflow: the active LaNiC_2 seed-scan tables use an 88-spinor, 245-translation-block survey Hamiltonian, while the active LaNiGa_2 survey Hamiltonian has 176 spinor Wannier functions and 245 translation blocks. These are orthorhombic LaNiX_2 inputs, so the material-derived sections use the Wannier Hamiltonian symmetry as supplied and should not be read as C4- or tetragonal-symmetric models. The workflow for these inputs is Quantum ESPRESSO, `pw2wannier90`, Wannier90, import of `wannier90_hr.dat`, a shift to the QE Fermi reference, and then the reduced- and full-basis INT diagnos-

TABLE II. QE/Wannier90 material inputs used by, or retained next to, the survey calculations. Here N_b/N_W is the number of QE bands and spinor Wannier functions, and spread is the final Wannier90 total spread in square Angstrom. Large spreads indicate provisional survey bases rather than production-quality interpolations. The LaNiC_2 full-window row is a completed improved candidate basis; its bounded rerun is discussed in the text, while the retained figure tables below still use the active 245-block LaNiC_2 survey row.

Input	SCF/NSCF mesh	N_b/N_W	spread
LaNiC_2 SOC	$8^2 \times 6 / 6^2 \times 4$	320/88	206.16
LaNiGa_2 SOC	$6^2 \times 4 / 6^2 \times 4$	360/176	929.75
LaNiC_2 SOC full-window	$10^2 \times 8 / 8^2 \times 6$	320/88	33.78 [†]

[†]Wannier90 reached `dis_num_iter`=800 without satisfying the disentanglement convergence criterion.

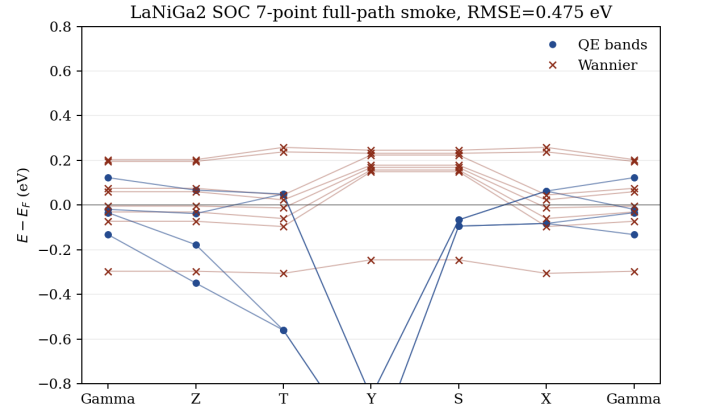


FIG. 10. LaNiGa_2 SOC Wannier overlay survey check. QE-direct bands and the Wannier interpolation are overlaid on the staged 25-k-point segmented high-symmetry-path calculation near E_F . The comparison has RMSE 0.472 eV and the underlying QE bands runs retain residual `c_bands` non-convergence warnings; a stricter cleanup pass converged only the first two segments before failing on $T-R$. The overlay is therefore provenance for the survey Hamiltonian rather than production-quality validation.

tics. The result is simple: the available material-derived Hamiltonians do not remove the obstruction. Reduced scans select unitary-INT-like branches with nonunitarity at numerical-noise scale, while first full-basis quick scans select onsite-singlet-like seeds above the normal reference. The detailed validation caveats are collected in Sec. VII; the material survey supports the conservative mechanism conclusion but does not carry the burden of proving a production LaNiX_2 ground state.

The compact reduced-basis benchmark first keeps four near-Fermi states. In this reduced model the LaNiC_2 SOC survey has moderate INT projection and a unitary-INT-like best seed, while the dense LaNiGa_2 SOC input also selects a unitary-INT-like branch in the four-state scan but remains nonunitary only at numerical-

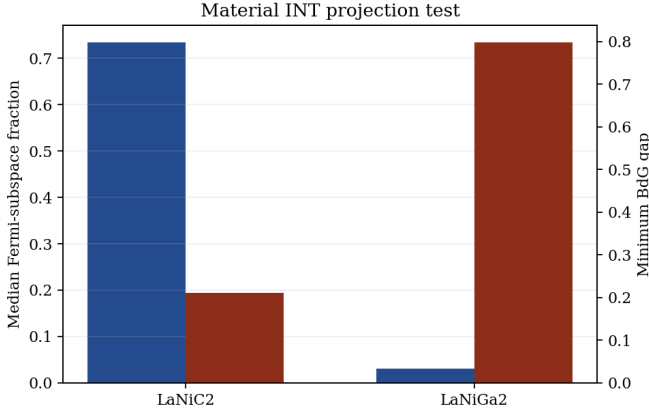


FIG. 11. Reduced material INT projection. The active dense LaNiGa₂ SOC reduction has weak INT Fermi-subspace projection in the current four-state truncation, and projection alone does not determine the relaxed phase.

TABLE III. Reduced material benchmark. The free-energy differences $\Delta\Omega = \Omega_{\text{MF}} - \Omega_N$ are relative to the normal reference in the compact scan; “u-INT” denotes a unitary INT seed.

Material	R	P	seed	$\Delta\Omega$	q
LaNiC ₂ SOC	245	0.735	u-INT	4.68e-6	1.16e-6
LaNiGa ₂ SOC	245	0.031	u-INT	1.37e-5	2.14e-6

noise scale. In both cases the best nonunitarity is at numerical-noise scale. In Tables III and IV, R is the number of retained Wannier90 real-space translation blocks in the imported Hamiltonian, P_{INT} is the median retained INT Fermi-subspace projection in the reduced scan, N is the active spinor Wannier-basis dimension, and q is the dimensionless scalar-identity deviation of $\Delta\Delta^\dagger$. Values $q \sim 10^{-6}$ are numerical-noise scale in these scans.

The code uses the orbital-rotation-invariant local Kanamori defaults $U' = U - 2J_H$ and $J_P = J_H$ unless a scan overrides them. The seed families are onsite singlet, unitary INT, nonunitary INT, mixed singlet-INT, and one random complex seed; reduced material scans keep four near-Fermi states, while the full-basis checks keep the 88-spinor LaNiC₂ and 176-spinor LaNiGa₂ Wannier bases.

Normal feedback was then included at three levels: anomalous-only BdG, density-density Kanamori Hartree feedback, and unrestricted compact Kanamori Hartree/Fock feedback. In the anomalous-only update, the Gor’kov contraction $\chi_{\ell_1\ell_2} = \langle c_{-\mathbf{k}\ell_2} c_{\mathbf{k}\ell_1} \rangle$ updates only the pairing field Δ . In the density-density Hartree update, the diagonal normal density $\rho_{\ell\ell} = \langle c_\ell^\dagger c_\ell \rangle$ updates direct density shifts in the normal field ϕ . In the unrestricted compact Kanamori update, the local sparse Hubbard-Kanamori vertex uses the compact normal density matrix ρ to update Hartree and Fock contributions to ϕ , while χ updates Δ . The compact tests include local Hartree/Fock exchange in the reduced four-state

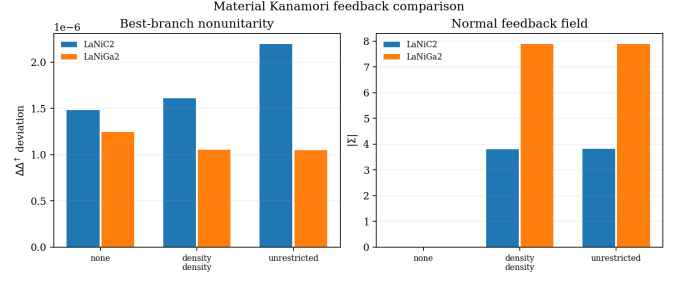


FIG. 12. Kanamori feedback comparison in the reduced material basis. Including normal-field feedback changes the compact free-energy landscape but does not stabilize a robust nonunitary INT branch.

basis; the first full-basis scans below remain deliberately quick anomalous scans and should not be read as production full-Wannier unrestricted exchange calculations. The normal feedback changes free energies and can move the best seed label, but it does not generate a clean nonunitary INT solution in the current material basis.

We also tested whether material normal-state response structure alone can supply the missing momentum dependence. On a reduced LaNiC₂ SOC Wannier basis retaining eight states from the 88-spinor survey Hamiltonian, the code computes finite- q Lindhard responses for diagonal density, spin-tag, and orbital-tag proxy operators. The strongest generic quick response is the identity-density proxy at (π, π) . Feeding the corresponding normalized response envelope into the k-resolved HFG bridge does not select a nonunitary INT branch in the three-iteration seed competition; the best short-run seed remains a non-INT competitor. Repeating the bridge after excluding the generic density response and keeping only the spin/orbital proxy operators also remains negative: the strongest focused response is the orbital-staggered proxy at $(\pi, 0)$, but the same short-run seed competition still relaxes away from nonunitary INT. These tests are useful negative evidence against simple material-response envelopes, but they are not a screened material four-fermion vertex. A stronger material mechanism would require constructing and testing the screened interaction tensor in the Wannier basis, rather than multiplying the pairing channel by a scalar Lindhard envelope. As a first tensor-bookkeeping step, the same response rows have now been assembled into an antisymmetric pair-space tensor on the reduced eight-state LaNiC₂ basis. The tensor has 28 antisymmetric pair states, satisfies the tested Hermiticity and left/right antisymmetry identities to numerical precision, and has eigenvalues spanning approximately $[-5.6 \times 10^{-2}, 5.1 \times 10^{-3}]$ for unit response scale. Projecting this quick tensor onto the normalized local channels gives the most attractive matrix element in the unitary-INT channel, about -2.53×10^{-2} for unit response scale. A companion three-iteration local HFG bookkeeping smoke driven directly by this tensor does

not select nonunitary INT; its lowest short-run seed is mixed singlet-INT and is not converged. These checks verify signs, pair counting, channel projection, and solver plumbing for a material-basis tensor representation, but the tensor still uses diagonal Lindhard proxy operators rather than cRPA matrix elements and is not a production self-consistent material interaction.

The retained figure settings are as follows. We reserve $\mathbf{U}$ for the bare local Kanamori interaction tensor. The self-consistency scans distinguish the bare local Kanamori tensor $\mathbf{U} = (U, U', J_H, J_P)$ from the attractive vertex used in a normalized anomalous channel. For the retained diagnostic scans the code records an effective INT attraction, equivalent to $\mathbf{U}_\alpha^{\text{eff}} = -|U_{\text{INT}}^{\text{eff}}|$, rather than identifying the pairing scale with the bare repulsive $V_{\text{INT}} = U' - J_H$ of Eq. (9). Legacy figure parameters still quote this positive channel magnitude as g_{pair} . The retained two-orbital toy normal state uses Eq. (3) with $t = 1$, $\mu = -2.7$, $s = 0.1$, $v_0 = 0.15$, $v_1 = 0$, and baseline longitudinal spin-orbit coupling $\lambda_z = 0.1$. The baseline projection diagnostic sets the transverse repair fields to zero; the displayed repaired toy example turns on $\lambda_x \tau_y \sigma_x$ with $\lambda_x = 0.6$. The older legacy three-dimensional diagnostic scripts used orbital-resolved hoppings $(t_a, t_b, t_{az}, t_{bz}) = (1.0, 0.7, 0.2, 0.2)$ and orbital chemical potentials $(\mu_a, \mu_b) = (1.0, 0.6)$; those values are retained in the repository for provenance but are not the active normal state used for the main retained figures. The toy HFG convergence uses effective channel magnitude $g_{\text{pair}} = 2.8$, $U = 3.0$, $J_H = 0.6$, temperature 0.03, a 3×3 mesh, $\gamma = 0.35$, and 8 iterations. The reduced material figures use effective INT attraction $|U_{\text{INT}}^{\text{eff}}| = 1.6$, an imposed Fermi window of 3.0 eV, a 5×5 projection grid, and a 2×2 HFB mesh for 5 iterations. The feedback comparison uses the same pairing scale with $U = 3.0$, $J_H = 0.6$, a 3×3 HFB mesh, $\gamma = 0.3$, and tolerance 10^{-7} . The first full-basis scan uses $g_{\text{pair}} = 1.2$, a 1×1 HFB mesh, $\gamma = 0.35$, and tolerance 10^{-6} . For material-derived calculations the normal-state hoppings are read directly from the corresponding Wannier90 real-space Hamiltonian files rather than from the toy parameters. All material Hamiltonians are shifted so that the chosen QE Fermi reference is at zero energy, i.e. the working chemical potential is $\mu = 0$ after this shift; the quick scans do not impose a separate fixed-filling constraint beyond that chemical-potential reference.

Finally, the same seed scan was repeated in the full active Wannier bases. This removes the most aggressive four-state truncation. In the first full-basis quick scan, the best relaxed superconducting seed is onsite-singlet-like rather than INT-like for both active materials; however, that seed remains above the normal reference in the quick free-energy estimate, and its nonunitarity is at numerical-noise scale.

TABLE IV. First full-basis material seed scan. For both active spin-orbit-coupled survey Hamiltonians, the lowest relaxed superconducting seed in this quick scan is onsite-singlet-like rather than INT-like. Positive $\Omega - \Omega_N$ means that this best superconducting seed is still above the normal reference in this estimate.

Material	N	seed	$\Omega - \Omega_N$	q
LaNiC ₂ SOC	88	onsite singlet	5.76×10^{-5}	9.45×10^{-7}
LaNiGa ₂ SOC	176	onsite singlet	5.54×10^{-6}	8.99×10^{-8}

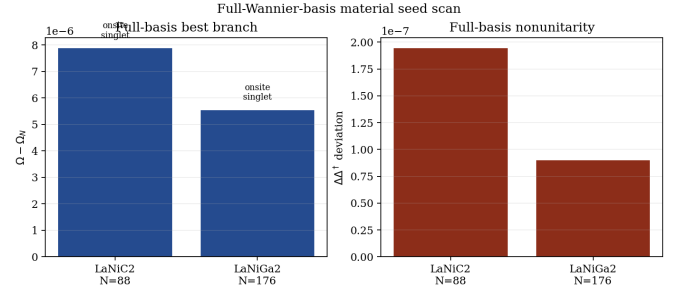


FIG. 13. Full-basis material seed scan. Keeping the full active Wannier basis removes the reduced four-state truncation. Within this quick superconducting-seed comparison the best relaxed seed is onsite-singlet-like for both LaNiC₂ and LaNiGa₂ SOC survey Hamiltonians, but Table IV shows that it remains above the normal reference.

VII. LIMITATIONS

The present result should not be overread. The material Hamiltonians used in the seed tables are survey Wannier models, not production-quality interpolations. This is especially important for LaNiGa₂: a staged 25-k-point segmented QE band comparison completed, but the overlay remains poor near E_F and the QE logs report residual `c_bands` nonconvergence in every segment. LaNiC₂ now has an improved full-window candidate HR file, but its disentanglement did not converge before the iteration cap and the checkpointed reduced/full-basis INT reruns through $n_k = 3$ still relax to numerical-zero order parameter rather than selecting nonunitary INT. The AMN audit further shows that raw trial-projector weights should not be interpreted as the literature partial DOS; a direct QE `projwfc.x` PDOS validation is currently running as Icarus job 9012215. The full interaction feedback is also incomplete at production scale. The compact tests include unrestricted Kanamori feedback, but the full-basis scans are deliberately quick anomalous-first calculations rather than production full-Wannier unrestricted Hartree-Fock-Gor'kov minimizations. The evidence-gate tests record that the bundled validation, full-basis, and Stoner benchmark paths execute, but they do not change the production status: in the tiny full-basis check both active materials have onsite-singlet-like best superconducting seeds above the normal reference, and the compact Stoner-coupled INT feedback decays.

TABLE V. Reviewer-facing status of the material evidence. The table separates negative self-consistency evidence, workflow blockers, and mechanism prototypes.

Evidence item	Current status	Interpretation
LaNiC ₂ survey	negative scans	diagnostic only
LaNiC ₂ full-window	negative $n_k \leq 3$; PDOS running	candidate only
LaNiGa ₂ SOC	validation blocked	rebuild route
Spin feedback	Stoner proxy	no material tensor
Pair tensor	signs checked	not cRPA
Composite selector	reduced target	not yet HFG

The first staged LaNiGa₂ full-path validation follow-up failed in QE’s `splitwf` routine with `wrong size for pwt`. A later minimal-checkpoint segmented serial-CG run completed and generated the 25-k-point comparison payload shown in Fig. 10, but its convergence warnings and large mismatch keep it below the standard needed for a strong material claim. A stricter cleanup pass converged the first two segments but failed on the T – R segment, so it is retained only as diagnostic evidence. A final $N_b = 360$ Γ – Z rerun completed and is bundled as a narrow band-count diagnostic only. What has been completed is a linearized local-channel diagnostic, imposed BdG algebraic and spectral diagnostics, toy and compact HFG feedback tests, and quick reduced/full-basis material surveys. What has not been completed is a production full-Wannier unrestricted nonlinear HFG minimum with validated material bands. We do not treat that missing calculation as an unfinished part of the present paper. Instead it defines a separate material-realistic follow-up program: a production-quality SOC Wannier basis, a clean full-path DFT/Wannier overlay near E_F , production fixed-filling control beyond the first smokes, a material-derived screened interaction tensor or otherwise justified full-basis interaction feedback calculation, and robustness against interaction, seed, mesh, and Wannier-window choices.

Table V summarizes the present claim readiness.

The promotion gates are correspondingly explicit. LaNiC₂ requires converged disentanglement/localization, a clean DFT–Wannier overlay in the active window, fixed-filling control, and a finite-gap branch that remains below normal and competing seeds. LaNiGa₂ first requires a `c_bands`-clean normal-state workflow. The stronger mechanism route requires a screened local or k -resolved interaction kernel that supports finite INT amplitude and selects finite $q_z = (\mathbf{id} \times \mathbf{d}^*)_z$ while preserving the degeneracy of the two zero-field time-reversed domains. Several finite- Q , pair-spin, and screened-fluctuation proxy kernels were tried in QuLab and are recorded there as mechanism-development diagnostics. They either collapse under convergence controls or remain explicitly designed positive controls. The paper therefore does not promote any of them to a microscopic mechanism.

These limitations determine the correct interpretation.

The calculations do not prove that INT is impossible in LaNiGa₂. They show that the specific route tested here—local Hubbard-Kanamori-like interactions applied to the current LaNiX₂ survey Hamiltonians—does not stabilize robust self-consistent nonunitary INT order. The next decisive calculations are a validated normal-state material basis and a physically justified screened pair-spin-selection kernel, not larger scalar-INT scans of the same collapsed branch.

VIII. CONCLUSION

INT pairing remains a coherent and physically diagnostic channel. It is naturally connected to Hund-favored interorbital triplet pairing, and an imposed nonunitary INT gap gives the expected split $\Delta\Delta^\dagger$, spin-resolved density of states, and condensate spin polarization. The self-consistency tests expose the restrictive step. Scalar INT attraction drives the amplitude coordinate A but does not by itself select the nonunitary imbalance coordinate Q . In the controlled toy calculations this produces finite INT amplitude without robust spontaneous nonunitarity; in the current material-derived LaNiC₂/LaNiGa₂ survey Hamiltonians the relaxed branches are unitary or onsite-singlet-like, with nonunitarity at numerical-noise scale.

The defensible conclusion is therefore a self-consistency obstruction, not a broad no-go theorem. The present calculations validate the INT ansatz as a channel and observable diagnostic, but they do not derive it as a robust self-consistent nonunitary ground state under the tested local Hubbard-Kanamori-like assumptions. A stronger microscopic mechanism would need an interaction channel that couples to pair-spin imbalance, such as exchange or spin-polarization feedback, while preserving the degeneracy of the two time-reversed domains. The material-specific LaNiX₂ program remains valuable, but it is a separate follow-up requiring production-quality Wannier validation and a justified screened interaction tensor. The same obstruction gives a useful design principle for later TRSB mechanisms: keep the superconducting amplitude from starving, then supply a source-free selector for the TRSB coordinate. The loop-supercurrent program follows this route in a different order-parameter space, replacing pair-spin imbalance by phase/current winding while retaining the central lesson that amplitude support and TRSB selection are separate self-consistency problems.

Appendix A: Stoner Mean-Field Benchmark

The Stoner comparison in Fig. 14 is an implementation benchmark, not an additional material claim. It verifies that the density-channel Stoner convention used in the code reproduces the expected spontaneous magnetization before the same feedback logic is tested in the INT condensate-coupled loop. The reference den-

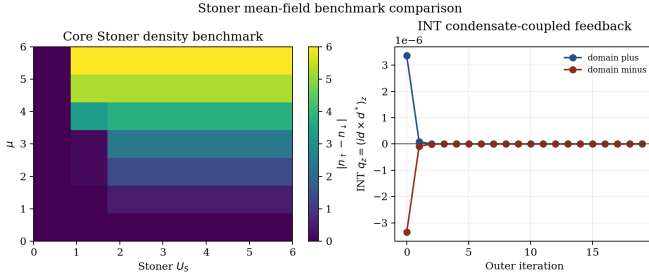


FIG. 14. Stoner mean-field benchmark comparison. Left: density-channel Stoner benchmark, following the Whittlesea thesis implementation [13], obtained by minimizing Eq. (A1); the magnetic region in the (U_S, μ) plane confirms the implemented sign convention. Right: INT condensate-coupled feedback test from Fig. 7; the condensate spin proxy relaxes to zero for the tested toy parameters. The comparison shows that the negative INT feedback result is not caused by a failure of the ordinary Stoner benchmark, but by the specific condensate-coupled mechanism tested here.

sity functional follows the Whittlesea thesis benchmark [13],

$$F_{\text{Stoner}}(n_{\uparrow}, n_{\downarrow}) = \alpha \left(n_{\uparrow}^{5/3} + n_{\downarrow}^{5/3} \right) - \mu(n_{\uparrow} + n_{\downarrow}) + U_S n_{\uparrow} n_{\downarrow}. \quad (\text{A1})$$

The scan minimizes Eq. (A1) over spin-resolved densities and reports the resulting magnetization proxy. The comparison separates two questions: whether the ordinary density Stoner benchmark magnetizes with the implemented sign convention, and whether the particular INT condensate-coupled feedback tested in Fig. 7 stabilizes a nonzero $id \times d^*$ fixed point. The first check is positive, while the second remains negative for the tested toy parameters.

Appendix B: Reproducibility and provenance

The retained figures are generated from the QuLab INT module and copied into the paper repository using the explicit mapping in the accompanying figure-provenance file. The publication Makefile contains targets to regenerate the QuLab figure set, copy the mapped

figures into the paper directory, and rebuild the PDF. The material input files are the LaNiC₂ Zhang-2018 SOC and LaNiGa₂ full-SOC Wannier90 Hamiltonians stored with their QE and Wannier90 input/output manifests. For LaNiGa₂, the validation overlay uses the staged 25-k-point segmented band-comparison payload when available; the older seven-point and partial Γ -Z overlays remain archived only as provenance. The evidence-gate audit still classifies the material inputs as survey-level rather than production-ready. For LaNiGa₂, the immediate blocker is normal-state validation: repeated QE `c_bands` failures and the rejected PAW-SOC route prevent promotion to a production material Hamiltonian on the present workflow. For LaNiC₂, the full-window Wannier-only continuation has now completed and provides an improved candidate HR file; checkpointed reduced/full-basis reruns on that basis through $n_k = 3$ are negative for nonunitary INT. The current AMN-vs-literature orbital-character discrepancy is being checked with the direct QE `projwfc.x` PDOS job 9012215; its result will inform a later material-realistic study, but the present paper does not depend on promoting the LaNiX₂ inputs beyond survey status. The reduced LaNiC₂ material-response envelope tests in `material_realistic_int_program/` are also negative, both for the generic identity-density response and for the focused spin/orbital proxy response. The first reduced pair-space tensor payload in the same directory verifies the antisymmetric tensor bookkeeping, and its companion three-iteration tensor-HFG smoke remains negative for nonunitary INT, but both are built from proxy Lindhard operators rather than ab-initio screened matrix elements. Production full-path DFT/Wannier validation, fixed filling or explicit chemical-potential control, a screened material interaction tensor suitable for full-basis HFG feedback, and broader robustness scans remain outstanding.

ACKNOWLEDGMENTS

The numerical workflows and figures were generated from the QuLab research code base [14].

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